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The transition form factors for semi-leptonic weak decays of J/ψ in QCD sum rules

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Abstract. Within the standard model, we investigate the semi-leptonic weak decays of J/ψ. The various form factors of J/ψ making the transition to a single charmed meson ($D^{(*)}_{(d,s)}$) are studied in the framework of QCD sum rules. These form factors fully determine the rates of the weak semi-leptonic decays of J/ψ and provide valuable information on non-perturbative QCD effects. Our results indicate that the decay rate of the semi-leptonic weak decay mode $J/\psi \to D_s^{(*)-} + e^+ + \nu_e$ is at the order of 10^{-10}.

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1 Introduction

Although strong and electromagnetic decays of J/ψ have been extensively studied for several decades, both experimental and theoretical investigations of weak decays of J/ψ are lagging much behind. Due to the smallness of the strength of the weak interaction, the weak decays of the J/ψ are rare processes. Sanchis-Lonzano suggested to search for these rare decays, whose sum of branching ratios were estimated to be at the order of 10^{-8} [1]. Such processes hardly drew much attention, because the database was far from reaching such accuracy. Thus, for a long time, few further researches on this topic were done. Thanks to the progress of accelerator and detector techniques, more accurate measurements may now be carried out, and thus the interest on weak decays of J/ψ has been revived. The BES collaboration indeed has started to measure some rare weak decays of J/ψ to eventually set an upper bound on the branching ratio of $J/\psi \to D + e + \nu_e$ at the order of 10^{-5} by using a 5.8×10^7 J/ψ database [2]. The forthcoming upgraded BESIII can accumulate 10^{10} J/ψ per year [3], which makes it marginally possible to measure such weak decays of J/ψ; at least one may expect to observe such events. Thus, a more careful theoretical investigation of these decays seems necessary.

Indeed, the weak decays of heavy quarkonium like J/ψ offer an ideal opportunity of studying non-perturbative QCD effects, because such systems contain two heavy constituents of the same flavor. The situation is quite different from that for heavy mesons that contain only one heavy constituent, and the non-perturbative effects might be attributed to the light flavor; thus heavy-quark effective theory (HQET) applies. Moreover, for the weak decay of a vector meson, the polarization effect may play a role in probing the underlying dynamics and hadron structure [1].

The weak decay of J/ψ is realized via the spectator mechanism in which the charm quark (antiquark) decays and the antiquark (quark) acts as a spectator. The characteristic of the decay modes is that the final state contains a single charmed hadron. The theory of weak interactions has been thoroughly investigated and the effective Hamiltonian at the quark level is perfectly formulated. The main job of calculating the rates of the semi-leptonic decays of J/ψ is to properly evaluate the hadronic matrix elements for $J/\psi \to D^{(*)}$, namely the transition form factors, which are obviously governed by non-perturbative QCD effects. The main aim of this work is to calculate the $J/\psi \to D^{(*)}_{(d,s)}$ form factors in the QCD sum rule approach.

The weak decay of heavy quarkonium has been studied by heavy quark spin symmetry [1]. In that framework, the transition form factors of a heavy quarkonium to heavy pseudoscalar and vector mesons are parameterized by a universal function $\eta_{12}(v_1 \cdot v_2)$ in analogy to the Isgur–Wise function for the heavy meson transitions. However, the non-recoil approximation $\eta_{12}(v_1 \cdot v_2) \approx 1$ was used in [1], which would bring about uncontrollable uncertainties to the estimation of decay widths. It seems helpful to re-investigate these processes based on a more rigorous theoretical framework. Motivated by the arguments, in this work we will calculate the form factors for heavy quarkonium J/ψ decays into a pseudoscalar or vector meson in the QCD sum rule approach.

As a matter of fact, many authors have tried to evaluate the transition form factors for the heavy meson and

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quarkonium system in various approaches, such as the simple quark model [4], the light-front approach [5–16], the QCD sum rule approach [17, 18], the perturbative QCD approach [19–24] etc. The QCD sum rule approach, which is rooted in quantum field theory and which is fully relativistic, is considered to be one of the effective tools for analyzing hadronic processes [17]. Besides evaluation of hadron spectra, the QCD sum rule technique has been applied to calculation of the pion electromagnetic form factor at intermediate momentum transfer [25–27], various weak decay channels [28],[1] the coupling constant of the strong interaction [30] and even to determine the light cone distribution amplitudes of hadrons [31–37]. The advantage of this method is that the non-perturbative QCD effects are included in a few parameters such as the quark and gluon condensates, which have an evident physical meaning [38].

After this introduction, we will first display the effective Hamiltonian relevant to the semi-leptonic decays of J/ψ to $D^{(*)-}_{d(s)}$, and the sum rules for the form factors in Sect. 2. The Wilson coefficients of various operators, which make manifest the perturbative QCD effects, are also calculated in this section with the help of the operator product expansion (OPE) technique. The numerical analysis of the form factors is performed in Sect. 3. The decay rates of the semi-leptonic decay $J/\psi \to D^{(*)-}_{d(s)} l^+\nu$ and a comparison of our results with that obtained based on other approaches are presented in Sect. 4. In the last section we draw our conclusions.

2 $J/\psi \to D^{(*)}_{d(s)}$ transition form factors in the QCD sum rule approach

2.1 Definitions of the $J/\psi \to D^{(*)}_{d(s)}$ transition form factors

For the semi-leptonic decays $J/\psi \to D^{(*)}_{d(s)} l^+\nu_l$, the effective weak Hamiltonian is given by

$$\mathcal{H}_{\rm eff}(c\to s(d)l\bar{\nu}_l) = \frac{G_{\rm F}}{\sqrt{2}} V^*_{cs(d)} \bar{s}(\bar{d})\gamma_\mu(1-\gamma_5)c\bar{\nu}_l\gamma^\mu(1-\gamma_5)l\,. \tag{1}$$

In calculating the rate of a semi-leptonic decay, the essential ingredient is the hadronic matrix element $\langle D^{(*)}_{d(s)}|\bar{s}\gamma_\mu(1-\gamma_5)c|J/\psi\rangle$, which is parameterized by various form factors [39]:

$$\begin{aligned}
&\langle D_{d(s)}(p_2)|\bar{q}\gamma_\mu(1-\gamma_5)c|J/\psi(\epsilon,p_1)\rangle \\
&= -\epsilon_{\mu\nu\alpha\beta}\epsilon^\nu p_1^\alpha p_2^\beta \frac{2V(q^2)}{m_\psi+m_D} \\
&\quad + \mathrm{i}(m_\psi+m_D)\left[\epsilon_\mu - \frac{\epsilon\cdot q}{q^2}q_\mu\right]A_1(q^2) \\
&\quad + \mathrm{i}\frac{\epsilon\cdot q}{m_\psi+m_D}A_2(q^2)\left[(p_1+p_2)_\mu - \frac{m_\psi^2-m_D^2}{q^2}q_\mu\right] \\
&\quad + 2\mathrm{i}m_\psi\frac{\epsilon\cdot q}{q^2}q_\mu A_0(q^2)\,,
\end{aligned} \tag{2}$$

$$\begin{aligned}
&\langle D^*_{d(s)}(\epsilon_2,p_2)|\bar{q}\gamma_\mu(1-\gamma_5)c|J/\psi(\epsilon_1,p_1)\rangle \\
&= -\mathrm{i}\epsilon_{\mu\nu\alpha\beta}\epsilon_1^\alpha\epsilon_2^{*\beta}\left[\left(p_1^\nu+p_2^\nu - \frac{m_\psi^2-m_{D^*}^2}{q^2}q^\nu\right)\widetilde{A}_1(q^2)\right. \\
&\quad \left. + \frac{m_\psi^2-m_{D^*}^2}{q^2}q^\nu\widetilde{A}_2(q^2)\right] \\
&\quad + \frac{\mathrm{i}}{m_\psi^2-m_{D^*}^2}\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta\left[\widetilde{A}_3(q^2)\epsilon_1^\nu\epsilon_2^*\cdot q - \widetilde{A}_4(q^2)\epsilon_2^{*\nu}\epsilon_1\cdot q\right] \\
&\quad + (\epsilon_1\cdot\epsilon_2^*)\left[-(p_{1\mu}+p_{2\mu})\widetilde{V}_1(q^2) + q_\mu\widetilde{V}_2(q^2)\right] \\
&\quad + \frac{(\epsilon_1\cdot q)(\epsilon_2^*\cdot q)}{m_\psi^2-m_{D^*}^2}\left[\left(p_{1\mu}+p_{2\mu} - \frac{m_\psi^2-m_{D^*}^2}{q^2}q_\mu\right)\widetilde{V}_3(q^2)\right. \\
&\quad \left. + \frac{m_\psi^2-m_{D^*}^2}{q^2}q_\mu\widetilde{V}_4(q^2)\right] \\
&\quad - (\epsilon_1\cdot q)\epsilon^*_{2\mu}\widetilde{V}_5(q^2) + (\epsilon_2^*\cdot q)\epsilon_{1\mu}\widetilde{V}_6(q^2)\,,
\end{aligned} \tag{3}$$

where the convention $\mathrm{Tr}[\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5] = 4\mathrm{i}\epsilon_{\mu\nu\rho\sigma}$ is adopted. For a transition of J/ψ into a charmed pseudoscalar meson that is induced by the weak current, there are four independent form factors: V, A_0, A_1 and A_2; while there are ten form factors for J/ψ making a transition to a charmed vector meson, which are parameterized as $\widetilde{A}_i$ $(i=1,2,3,4)$*and* $\widetilde{V}_j$ $(j=1,2,3,4,5,6)$. It is worthwhile to emphasize that the parametrization of the hadronic matrix element for J/ψ making a transition to a vector meson given in (3) has been studied less before. A similar matrix element for the transition of a vector to another vector meson that is induced by the electromagnetic current was investigated by Kagan in [40].

2.2 The transition form factors in the QCD sum rule approach

In this subsection, we calculate the transition form factors of $J/\psi \to D^{(*)-}_{(d,s)}$ by QCD sum rules. Here we present the formulations for the $J/\psi \to D_s^{(*)-}$ transition explicitly, while the expressions for $J/\psi \to D^{(*)-}$ can be obtained by the simple replacements of $D_s^{(*)-} \to D^{(*)-}$ and s quark to d quark.

2.2.1 The matrix element for $J/\psi \to D_s^-$

Following the standard procedure of the QCD sum rule approach [25, 26], we write the three-point correlation function for J/ψ making a transition to D_s^- as

$$\Pi_{\mu\nu} = \mathrm{i}^2\int \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{e}^{-\mathrm{i}p_1\cdot y+\mathrm{i}p_2\cdot x}\langle 0|j_5^{D_s^-}(x)j_\mu(0)j_\nu^{J/\psi}(y)|0\rangle\,, \tag{4}$$

where the current $j_\nu^{J/\psi}(y) = \bar{c}(y)\gamma_\nu c(y)$ represents the J/ψ channel; $j_\mu(0) = \bar{s}\gamma_\mu(1-\gamma_5)c$ is the weak current and

[1] For a review of QCD sum rule applications to weak decays of heavy mesons, see [29].

$j_5^{D^-}(x)=\bar{c}(x)\mathrm{i}\gamma_5 s(x)$ corresponds to the D_s^- channel. In terms of the definitions

$$\langle 0|\bar{c}\gamma_\nu c|J/\psi\rangle = m_\psi f_\psi \epsilon_\nu^\lambda\,, \quad \langle 0|\bar{c}\mathrm{i}\gamma_5 s|D_s\rangle = \frac{f_{D_s} m_{D_s}^2}{m_c+m_s}\,, \tag{5}$$

we can insert a complete set of hadronic states with the same quantum numbers as J/ψ and D_s^- to achieve the hadronic representation of the correlator (4):

$$\Pi_{\mu\nu} = \frac{f_{D_s} m_{D_s}^2 \langle D_s|j_\mu|J/\psi\rangle m_\psi f_\psi \epsilon_\nu^{*\lambda}}{\left(m_{J/\psi}^2-p_1^2\right)\left(m_{D_s}^2-p_2^2\right)(m_c+m_s)} + \text{contributions from higher states}\,. \tag{6}$$

Obviously the lowest hadronic states concerned are J/ψ and D_s, while the terms with "higher states" represent contributions coming from higher excited states and from the continuum. Using the double dispersion relation, the contributions of excited states and continuum can be expressed as

$$\text{contributions from higher states} = \iint_{\Sigma_{12}} \mathrm{d}s_1\,\mathrm{d}s_2 \frac{\rho_{\mu\nu}^h(s_1,s_2,q^2)}{\left(s_1-p_1^2\right)\left(s_2-p_2^2\right)} + \text{subtraction terms}\,, \tag{7}$$

where Σ_{12} denotes the integration region in the (s_1,s_2) plane. $\rho_{\mu\nu}^h$ is the spectral density at the hadron level. The subtraction terms are polynomials of either p_1 or p_2, which should disappear after performing a double Borel transformation $\widehat{\mathcal{B}}_{M_1^2}\widehat{\mathcal{B}}_{M_2^2}$, with

$$\widehat{\mathcal{B}}_{M_i^2} = \lim_{\substack{-p_i^2,n\to\infty \\ -p_i^2/n=M^2}} \frac{\left(-p_i^2\right)^{(n+1)}}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}p_i^2}\right)^n. \tag{8}$$

On the other side, we calculate the correlation function at the quark level by using the OPE to find

$$\Pi_{\mu\nu} = -f_0\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta - \mathrm{i}(f_1 p_{1\mu}p_{1\nu} + f_2 p_{2\mu}p_{2\nu} + f_3 p_{2\mu}p_{1\nu} + f_4 p_{1\mu}p_{2\nu} + f_5 g_{\mu\nu})\,, \tag{9}$$

where each coefficient contains contributions from both the perturbative part and the non-perturbative part, whose effects are manifest in several typical condensates,

$$f_i = f_i^{\text{pert}}\mathbf{I} + f_i^{qq}\langle\bar{q}q\rangle + f_i^{GG}\langle GG\rangle + f_i^{qGq}\langle\bar{q}Gq\rangle + \dots\,, \tag{10}$$

with f_i^{pert}, f_i^{qq}, f_i^{GG} and f_i^{qGq},... denoting the contributions to the correlation functions from dimension 0, 3, 4, 5, ... operators. By quark–hadron duality, one may match the two different representations of the correlation function and perform a double Borel transformation on the variables p_1 and p_2; then we get the sum rules for the form factors:

$$V(q^2) = -\frac{(m_c+m_s)(m_\psi+m_{D_s})}{2m_\psi f_\psi f_{D_s} m_{D_s}^2} \times \mathrm{e}^{m_\psi^2/M_1^2}\mathrm{e}^{m_{D_s}^2/M_2^2} M_1^2 M_2^2 \widehat{\mathcal{B}} f_0\,, \tag{11}$$

$$A_1(q^2) = \frac{(m_c+m_s)}{(m_\psi+m_{D_s})m_\psi f_\psi f_{D_s} m_{D_s}^2} \times \mathrm{e}^{m_\psi^2/M_1^2}\mathrm{e}^{m_{D_s}^2/M_2^2} M_1^2 M_2^2 \widehat{\mathcal{B}} f_5\,, \tag{12}$$

$$A_2(q^2) = -\frac{(m_c+m_s)(m_\psi+m_{D_s})}{2m_\psi f_\psi f_{D_s} m_{D_s}^2} \times \mathrm{e}^{m_\psi^2/M_1^2}\mathrm{e}^{m_{D_s}^2/M_2^2} M_1^2 M_2^2 \widehat{\mathcal{B}}(f_2+f_4)\,, \tag{13}$$

$$A_0(q^2) = -\frac{(m_c+m_s)}{2m_\psi^2 f_\psi f_{D_s} m_{D_s}^2}\mathrm{e}^{m_\psi^2/M_1^2}\mathrm{e}^{m_{D_s}^2/M_2^2} M_1^2 M_2^2 \times\left[\widehat{\mathcal{B}}(f_2+f_4)\frac{m_\psi^2-m_{D_s}^2}{2} - \widehat{\mathcal{B}}(f_2-f_4)\frac{q^2}{2} - \widehat{\mathcal{B}}f_5\right]. \tag{14}$$

2.2.2 The matrix element for $J/\psi \to D_s^{*-}$

The three-point correlation function of J/ψ to D_s^{*-} is

$$\Pi_{\mu\nu\rho} = \mathrm{i}^2\int \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{e}^{-\mathrm{i}p_1\cdot y+\mathrm{i}p_2\cdot x}\langle 0|j_\rho^{D_s^*}(x)j_\mu(0)j_\nu^{J/\psi}(y)|0\rangle\,, \tag{15}$$

where the current $j_\rho^{D_s^*}(x)=\bar{c}(x)\gamma_\rho s(x)$ denotes the D_s^{*-} channel, and $j_\nu^{J/\psi}(y)$ and $j_\mu(0)$ are defined as in the above subsection. One the one hand, inserting the hadron states, the correlation function is written as

$$\Pi_{\mu\nu\rho} = \frac{m_{D_s^*} f_{D_s^*}\epsilon_\rho'^{\lambda'}\langle D_s^*|j_\mu|J/\psi\rangle m_{J/\psi} f_{J/\psi}\epsilon_\nu^{*\lambda}}{\left(m_{J/\psi}^2-p_1^2\right)\left(m_{D_s^*}^2-p_2^2\right)} + \iint \mathrm{d}s_1\,\mathrm{d}s_2 \frac{\rho_{\mu\nu\rho}^h(s_1,s_2,q^2)}{\left(s_1-p_1^2\right)\left(s_2-p_2^2\right)} + \text{subtraction terms}\,. \tag{16}$$

On the other hand, the correlation function at the quark level is formulated as

$$\begin{aligned}\Pi_{\mu\nu\rho} =\; & \mathrm{i}F_1\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta p_{1\rho} + \mathrm{i}F_2\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta p_{2\rho} \\ & + \mathrm{i}F_3\epsilon_{\mu\rho\alpha\beta}p_1^\alpha p_2^\beta p_{1\nu} + \mathrm{i}F_4\epsilon_{\mu\rho\alpha\beta}p_1^\alpha p_2^\beta p_{2\nu} \\ & + \mathrm{i}F_5\epsilon_{\nu\rho\alpha\beta}p_1^\alpha p_2^\beta p_{1\mu} + \mathrm{i}F_6\epsilon_{\nu\rho\alpha\beta}p_1^\alpha p_2^\beta p_{2\mu} \\ & + F_7 g_{\mu\nu}p_{1\rho} + F_8 g_{\mu\rho}p_{1\nu} + F_9 g_{\nu\rho}p_{1\mu} \\ & + F_{10} g_{\mu\nu}p_{2\rho} + F_{11} g_{\mu\rho}p_{2\nu} + F_{12} g_{\nu\rho}p_{2\mu} \\ & + F_{13}p_{1\mu}p_{1\nu}p_{1\rho} + F_{14}p_{2\mu}p_{2\nu}p_{1\rho} + F_{15}p_{1\mu}p_{2\nu}p_{1\rho} \\ & + F_{16}p_{2\mu}p_{1\nu}p_{1\rho} + F_{17}p_{2\mu}p_{2\nu}p_{2\rho} + F_{18}p_{1\mu}p_{1\nu}p_{2\rho} \\ & + F_{19}p_{2\mu}p_{1\nu}p_{2\rho} + F_{20}p_{1\mu}p_{2\nu}p_{1\rho}\,,\end{aligned} \tag{17}$$

where each coefficient F_i includes contributions from both perturbative and non-perturbative parts and is written explicitly as

$$F_i = F_i^{\text{pert}}\mathbf{I} + F_i^{qq}\langle\bar{q}q\rangle + F_i^{GG}\langle GG\rangle + F_i^{qGq}\langle\bar{q}Gq\rangle + \dots \tag{18}$$

Again, equating the correlation functions calculated in these two frameworks and performing Borel transformations on both sides, we derive the form factors of $J/\psi \to D_s^{*-}$ as follows:

$$\widetilde{A}_1(q^2) = -\frac{1}{4m_\psi f_\psi m_{D^{*-}} f_{D_s^*}} \mathrm{e}^{m_\psi^2/M_1^2} \mathrm{e}^{m_{D_s^*}^2/M_2^2} M_1^2 M_2^2 \times \widehat{\mathcal{B}}\left[(F_5 - F_6)q^2 + (F_5 + F_6)\left(m_\psi^2 - m_{D_s^*}^2\right)\right], \tag{19}$$

$$\widetilde{A}_2(q^2) = \frac{m_{D_s^*}^4 - 2\left(q^2 + m_\psi^2\right)m_{D_s^*}^2 + \left(q^2 - m_\psi^2\right)^2}{4\left(m_{D_s^*}^2 - m_\psi^2\right) m_\psi f_\psi m_{D_s^*} f_{D_s^*}} \times \mathrm{e}^{m_\psi^2/M_1^2} \mathrm{e}^{m_{D_s^*}^2/M_2^2} M_1^2 M_2^2 \widehat{\mathcal{B}}(F_5 + F_6), \tag{20}$$

$$\widetilde{A}_3(q^2) = \frac{m_\psi^2 - m_{D_s^*}^2}{m_\psi f_\psi m_{D_s^*} f_{D_s^*}} \mathrm{e}^{m_\psi^2/M_1^2} \mathrm{e}^{m_{D_s^*}^2/M_2^2} M_1^2 M_2^2 \times \widehat{\mathcal{B}}(F_1 - F_5), \tag{21}$$

$$\widetilde{A}_4(q^2) = \frac{{m_\psi}^2 - m_{D_s^*}^2}{m_\psi f_\psi m_{D_s^*} f_{D_s^*}} \mathrm{e}^{m_\psi^2/M_1^2} \mathrm{e}^{m_{D_s^*}^2/M_2^2} M_1^2 M_2^2 \times \widehat{\mathcal{B}}(F_4 + F_6), \tag{22}$$

$$\widetilde{V}_1(q^2) = -\frac{1}{2m_\psi f_\psi m_{D_s^*} f_{D_s^*}} \mathrm{e}^{m_\psi^2/M_1^2} \mathrm{e}^{m_{D_s^*}^2/M_2^2} M_1^2 M_2^2 \times \widehat{\mathcal{B}}(F_9 + F_{12}), \tag{23}$$

$$\widetilde{V}_2(q^2) = \frac{1}{2m_\psi f_\psi m_{D_s^*} f_{D_s^*}} \mathrm{e}^{m_\psi^2/M_1^2} \mathrm{e}^{m_{D_s^*}^2/M_2^2} M_1^2 M_2^2 \times \widehat{\mathcal{B}}(F_9 - F_{12}), \tag{24}$$

$$\widetilde{V}_3(q^2) = \frac{m_{D_s^*}^2 - {m_\psi}^2}{2m_\psi f_\psi m_{D_s^*} f_{D_s^*}} \mathrm{e}^{m_\psi^2/M_1^2} \mathrm{e}^{m_{D_s^*}^2/M_2^2} M_1^2 M_2^2 \times \widehat{\mathcal{B}}(F_{14} + F_{15}), \tag{25}$$

$$\widetilde{V}_4(q^2) = \frac{1}{2m_\psi f_\psi m_{D_s^*} f_{D_s^*}} \mathrm{e}^{m_\psi^2/M_1^2} \mathrm{e}^{m_{D_s^*}^2/M_2^2} M_1^2 M_2^2 \times \widehat{\mathcal{B}}\left[(F_{14} - F_{15})q^2 + (F_{14} + F_{15})\left(m_{D_s^*}^2 - {m_\psi}^2\right)\right], \tag{26}$$

$$\widetilde{V}_5(q^2) = \frac{1}{m_\psi f_\psi m_{D_s^*} f_{D_s^*}} \mathrm{e}^{m_\psi^2/M_1^2} \mathrm{e}^{m_{D_s^*}^2/M_2^2} M_1^2 M_2^2 \widehat{\mathcal{B}} F_{11}, \tag{27}$$

$$\widetilde{V}_6(q^2) = \frac{1}{m_\psi f_\psi m_{D^{*-}} f_{D_s^*}} \mathrm{e}^{m_\psi^2/M_1^2} \mathrm{e}^{m_{D_s^*}^2/M_2^2} M_1^2 M_2^2 \widehat{\mathcal{B}} F_7. \tag{28}$$

2.3 The Wilson coefficients for the correlation function $\Pi_{\mu\nu}$

In this subsection we calculate the Wilson coefficients that are defined above. To guarantee sufficient theoretical accuracy, the correlation functions are required to be expanded up to dimension-5 operators, namely the quark–gluon mixing condensate. The dimension-6 operators, such as the four quark condensates, are small and are further suppressed by $O(\alpha_s^2)$, so they can safely be neglected in our calculations.

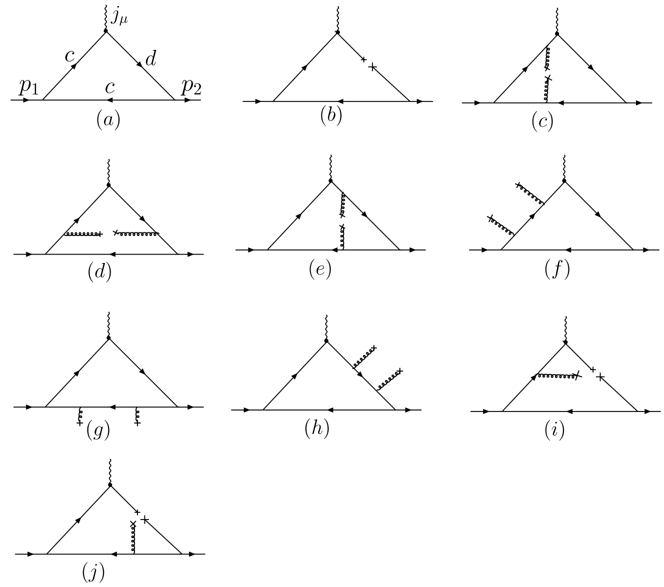

Fig. 1. Graphs for the Wilson coefficients in the operator product expansion of the correlation function. **a** is for the contribution of unit operator; **b** for the two-quark condensate; **c**–**h** describe the contributions from gluon condensate; **i** and **j** are for the quark–gluon mixing condensate

The diagrams that depict the contributions from the perturbative part and non-perturbative condensates are shown in Fig. 1. The first diagram results in the Wilson coefficient of the unit operator; the second diagram is relevant to the contribution of the quark condensate, where the heavy-quark condensate is neglected. The Wilson coefficient of the two-gluon condensate operator is obtained from Fig. 1c–h. The last two diagrams, Fig. 1i and j, stand for the contribution of a quark–gluon mixing condensate. In this work, all of the Wilson coefficients are calculated at the lowest order in the running coupling constant of the strong interaction.

2.3.1 Perturbative contributions to Wilson coefficients for $\Pi_{\mu\nu}$

The perturbative contribution to the three-point correlation function $\Pi_{\mu\nu}$ shown in Fig. 1a is included in the following amplitude:

$$C_{\mu\nu}^{\mathrm{pert}} = 3\mathrm{i}^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} (-1) \mathrm{Tr}\left[\gamma_\nu \frac{\mathrm{i}}{\not{k} - m_c} \mathrm{i}\gamma_5 \frac{\mathrm{i}}{\not{p}_2 + \not{k} - m_q} \times \gamma_\mu (1 - \gamma_5) \frac{\mathrm{i}}{\not{p}_1 + \not{k} - m_c}\right], \tag{29}$$

where m_q denotes the mass of the light quark in the D meson, and the factor “3” is due to the color loop. Using the dispersion relation, $C_{0\mu\nu}^{\mathrm{pert}}$ is written as

$$C_{0\mu\nu}^{\mathrm{pert}} = \iint \mathrm{d}s_1 \, \mathrm{d}s_2 \frac{\rho_{\mu\nu}^{\mathrm{pert}}\left(s_1, s_2, q^2\right)}{\left(s_1 - p_1^2\right)\left(s_2 - p_2^2\right)}. \tag{30}$$

The integration region is determined by the following condition:

$$-1 \le \frac{2s_1(s_2+m_c^2-m_q^2)-s_1(s_1+s_2-q^2)}{\lambda^{1/2}(s_1,s_2,q^2)\lambda^{1/2}(m_c^2,s_1,m_c^2)} \le 1\,, \quad (31)$$

where $\lambda(a,b,c)=a^2+b^2+c^2-2ab-2ac-2bc$. The standard way to calculate the spectral function $\rho_{\mu\nu}(s_1,s_2,q^2)$ is described here [25,26]: first, it is essential to calculate the double discontinuity of the amplitude, which can be realized by putting all the internal quark lines of Fig. 1a on their mass-shell and substituting the denominators of the quark propagators by the δ functions based on Cutkosky's cutting rule,

$$\frac{1}{k^2-m^2+\mathrm{i}\epsilon} \to -2\pi\mathrm{i}\delta(k^2-m^2)\,. \quad (32)$$

Then, the spectral function can easily be achieved. Finally, we get the expression of the spectral function in the following form:

$$\begin{aligned}\rho_{\mu\nu}(p_1^2,p_2^2,q^2) &= \frac{3}{(2\pi\mathrm{i})^2}(-2\pi\mathrm{i})^3\int\frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{Tr}[\gamma_\nu(\not k+m_c)\\ &\times\gamma_5(\not p_2+\not k+m_q)\gamma_\mu(1-\gamma_5)(\not p_1+\not k+m_c)]\\ &\times\delta(k^2-m_c^2)\delta[(p_2+k)^2-m_q^2]\delta[(p_1-k)^2-m_c^2]\,.\end{aligned} \quad (33)$$

After tedious calculations, one finally obtains the perturbative contribution to the correlation function, which can be decomposed as the sum of various terms according to different Lorentz structures, namely,

$$\begin{aligned}\rho_{\mu\nu}^{\mathrm{pert}} = &-\rho_0^{\mathrm{pert}}\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta - \mathrm{i}(\rho_1^{\mathrm{pert}}p_{1\mu}p_{1\nu}+\rho_2^{\mathrm{pert}}p_{2\mu}p_{2\nu}\\ &+\rho_3^{\mathrm{pert}}p_{2\mu}p_{1\nu}+\rho_4^{\mathrm{pert}}p_{1\mu}p_{2\nu}+\rho_5^{\mathrm{pert}}g_{\mu\nu})\,.\end{aligned} \quad (34)$$

The expressions for the ρ_i^{pert} are a bit more tedious, so we will display their explicit forms in Appendix A.

2.3.2 The quark condensate contribution

Now we turn to the calculation of the Wilson coefficient of the quark condensate operator, shown in Fig. 1b. One can easily find that it does not contribute to the correlation function after performing a double Borel transformation on both variables p_1^2 and p_2^2, since the propagator of this diagram, $\frac{1}{(p_2^2-m_c^2)(q^2-m_c^2)}$, only depends on the variable p_2^2. In other words, the Wilson coefficient of the dimension-3 two quark condensate turns out to be zero in the leading order of the heavy-quark mass expansion after carrying out a double Borel transformation. As can be seen, vanishing of the contributions from the quark condensate is independent on the structures of the effective vertices; therefore, it also does not contribute to the decays of J/ψ into a vector meson for the same reason. Below, we do not need to investigate the contributions of the quark condensate to $J/\psi \to D^*$ based on this argument.

2.3.3 The contribution from a gluon condensate

The diagrams which determine the Wilson coefficient of the gluon condensate are shown in Fig. 1c–h. The standard way is using the so-called fixed-point gauge technique. The gauge fixing condition is

$$x^\mu A_\mu^a = 0\,, \quad (35)$$

where A_μ^a is the gluon field. In the momentum space, $A_\mu^a(k)$ is transformed to the gauge invariant field strength by

$$A_\mu^a(k) = -\frac{\mathrm{i}}{2}(2\pi)^4 G_{\rho\mu}^a(0)\frac{\partial}{\partial k_\rho}\delta^4(k)+\ldots \quad (36)$$

Indeed, the loop integral

$$\begin{aligned}&I_{\mu_1,\mu_2,\ldots,\mu_n}(a,b,c)\\ &=\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{k_{\mu_1}k_{\mu_2}\cdots k_{\mu_n}}{[k^2-m^2]^a\left[(p_1+k)^2-m_1^2\right]^b\left[(p_2+k)^2-m_2^2\right]^c}\,,\end{aligned} \quad (37)$$

which is encountered in the work, is not easy to be performed by the Feynman parameter method. One alternative way to calculate this kind of integrals has extensively been discussed in [38,41–43], where the authors suggested to work in Euclidean space-time and employ the Schwinger representation for propagators. Instead, in our work, we follow the method employed in [44–46], namely, to directly calculate the imaginary part of the integrals in terms of Cutkosky's rule.

With the help of the Mathematica package "FeynCalc", we finally get the contributions of Fig. 1c–h at the price of some long and tedious derivations and time-consuming computer computations. The contributions of the gluon condensates from various sources cancel each other completely after carrying out a double Borel transformation to the variables p_1^2 and p_2^2. Therefore, the diagrams involving the gluon condensate do not contribute to the transition of a vector meson J/ψ to a pseudoscalar D meson. This argument also applies to the transition of a pseudoscalar meson to a vector as discussed in [44–46], since the topologies of the Feynman diagrams that result in the Wilson coefficient of the gluon condensate are the same. As analyzed later, it is also true for the transition of J/ψ to a vector meson. However, we find that the flavor-changing neutral current process can receive non-zero contributions from the gluon condensate. It should be noted that the null contributions of gluon condensates to sum rules for the weak transition $c \to s(d)$ are different from that obtained in [41–43], where the method they adopted does not allow for the subtraction of continuum contributions.

2.3.4 The quark–gluon mixing condensate contribution

Finally, we proceed by calculating the Wilson coefficients of the dimension-5 operator $\langle\bar{q}Gq\rangle$. Only the two diagrams shown in Fig. 1i and j are involved. Concentrating on these two diagrams, we find that they do not contribute to the correlation function, due to the same reason as that for the null contribution from the quark condensate, namely, only the variable p_2^2 appears in the propagators, and the amplitude will vanish due to a double Borel transformation.

As mentioned at the beginning of this section, we do not consider the four quark condensate; hence, only the perturbative part, which corresponds to Fig. 1a, offers a non-zero contribution to the correlation function.

2.4 The Wilson coefficients for the operators contributing to the correlation function $\Pi_{\mu\nu\rho}$

After the above lengthy discussions, a computation of the correlation function $\Pi_{\mu\nu\rho}$ that determines the transition amplitude of J/ψ to a vector meson is straightforward. Repeating the previous calculations, but replacing the vertex for the pseudoscalar meson to that for a vector meson, one can obtain the expressions of the Wilson coefficients for all the operators concerned.

2.4.1 The calculations of the perturbative contribution to $\Pi_{\mu\nu\rho}$

The Wilson coefficient of the perturbative part corresponding to Fig. 1a is

$$C_{\mu\nu\rho}^{\text{pert}} = 3\mathrm{i}^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4}(-1)\mathrm{Tr}\left[\gamma_\nu \frac{\mathrm{i}}{\not{k}-m_c}\gamma_\rho \frac{\mathrm{i}}{\not{p}_2+\not{k}-m_d} \times \gamma_\mu(1-\gamma_5)\frac{\mathrm{i}}{\not{p}_1+\not{k}-m_c}\right]. \tag{38}$$

We rewrite it in the form of dispersion integrals for the sake of connecting it to the hadronic spectral density based on the assumption of the quark–hadron duality, as follows:

$$C_{\mu\nu\rho}^{\text{pert}} = \iint \mathrm{d}s_1\,\mathrm{d}s_2 \frac{\rho_{\mu\nu\rho}^{\text{pert}}\left(s_1, s_2, q^2\right)}{\left(s_1-p_1^2\right)\left(s_2-p_2^2\right)}\,. \tag{39}$$

The integration region is the same as that for $C_{\mu\nu}^{\text{pert}}$, which is presented in (31). Setting all the internal quark lines on their mass-shells, we derive the spectral function $\rho_{\mu\nu\rho}^{\text{pert}}$ as follows:

$$\begin{aligned}
\rho_{\mu\nu\rho}^{\text{pert}} = {} & \mathrm{i}\rho_1'^{\text{pert}}\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta p_{1\rho} + \mathrm{i}\rho_2'^{\text{pert}}\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta p_{2\rho} \\
& + \mathrm{i}\rho_3'^{\text{pert}}\epsilon_{\mu\rho\alpha\beta}p_1^\alpha p_2^\beta p_{1\nu} + \mathrm{i}\rho_4'^{\text{pert}}\epsilon_{\mu\rho\alpha\beta}p_1^\alpha p_2^\beta p_{2\nu} \\
& + \mathrm{i}\rho_5'^{\text{pert}}\epsilon_{\nu\rho\alpha\beta}p_1^\alpha p_2^\beta p_{1\mu} + \mathrm{i}\rho_6'^{\text{pert}}\epsilon_{\nu\rho\alpha\beta}p_1^\alpha p_2^\beta p_{2\mu} \\
& + \rho_7'^{\text{pert}}g_{\mu\nu}p_{1\rho} + \rho_8'^{\text{pert}}g_{\mu\rho}p_{1\nu} + \rho_9'^{\text{pert}}g_{\nu\rho}p_{1\mu} \\
& + \rho_{10}'^{\text{pert}}g_{\mu\nu}p_{2\rho} + \rho_{11}'^{\text{pert}}g_{\mu\rho}p_{2\nu} + \rho_{12}'^{\text{pert}}g_{\nu\rho}p_{2\mu} \\
& + \rho_{13}'^{\text{pert}}p_{1\mu}p_{1\nu}p_{1\rho} + \rho_{14}'^{\text{pert}}p_{2\mu}p_{2\nu}p_{1\rho} \\
& + \rho_{15}'^{\text{pert}}p_{1\mu}p_{1\nu}p_{1\rho} + \rho_{16}'^{\text{pert}}p_{2\mu}p_{1\nu}p_{1\rho} \\
& + \rho_{17}'^{\text{pert}}p_{2\mu}p_{2\nu}p_{2\rho} + \rho_{18}'^{\text{pert}}p_{1\mu}p_{1\nu}p_{2\rho} \\
& + \rho_{19}'^{\text{pert}}p_{2\mu}p_{1\nu}p_{2\rho} + \rho_{20}'^{\text{pert}}p_{1\mu}p_{2\nu}p_{1\rho}\,.
\end{aligned} \tag{40}$$

Only $\rho_i'^{\text{pert}}$ $(i = 1, 4, 5, 6, 7, 9, 11, 12, 14, 15)$ are related to the form factors $\widetilde{V}_1$, $\widetilde{V}_2$, $\widetilde{V}_3$, $\widetilde{V}_4$, $\widetilde{V}_5$, $\widetilde{V}_6$, $\widetilde{A}_1$, $\widetilde{A}_2$, $\widetilde{A}_3$ and $\widetilde{A}_4$, and we display their expressions in Appendix B.

2.4.2 The contribution of the gluon condensate to $\Pi_{\mu\nu\rho}$

Similar to the derivation made above, we easily obtain the Wilson coefficient of the gluon condensate, which may contribute to the correlation function $\Pi_{\mu\nu\rho}$. Then we rewrite the Wilson coefficient in the form of dispersion integrals:

$$C_{\mu\nu\rho}^{GG} = \iint \mathrm{d}s_1\,\mathrm{d}s_2 \frac{\rho_{\mu\nu\rho}^{GG}\left(s_1, s_2, q^2\right)}{\left(s_1-p_1^2\right)\left(s_2-p_2^2\right)}\,, \tag{41}$$

where the integral region is the same as that for the perturbative part.

The Lorentz structures corresponding to $\rho_i^{(GG)}$ $(i = 1, 4, 5, 6, 7, 9, 11, 12, 14, 15)$ are

$$\begin{aligned}
\rho_{\mu\nu\rho}^{GG} = {} & \mathrm{i}\rho_1'^{GG}\epsilon_{\mu\nu\rho\lambda}p_1^\lambda + \mathrm{i}\rho_4'^{GG}\epsilon_{\mu\nu\rho\lambda}p_2^\lambda + \mathrm{i}\rho_5'^{GG}\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_1^\beta p_{1\nu} \\
& + \mathrm{i}\rho_6'^{GG}\epsilon_{\mu\rho\alpha\beta}p_1^\alpha p_1^\beta p_{1\rho} + \rho_7'^{GG}g_{\mu\nu}p_{1\rho} + \rho_9'^{GG}g_{\nu\rho}p_{1\mu} \\
& + \rho_{11}'^{GG}g_{\mu\rho}p_{2\nu} + \rho_{12}'^{GG}g_{\nu\rho}p_{2\mu} + \rho_{14}'^{GG}g_{\mu\rho}p_{2\nu} \\
& + \rho_{15}'^{GG}g_{\nu\rho}p_{2\mu} + \ldots
\end{aligned} \tag{42}$$

After some long and tedious calculations, we find that all of the above coefficients $\rho_i'^{GG}$ are zero. This is completely the same as for the $\Pi_{\mu\nu}$ case. Therefore, only the perturbative part survives after performing a double Borel transformation on the two variables p_1^2 and p_2^2 at the leading order of the heavy-quark mass expansion and QCD running coupling constant expansion for the three-point function $\Pi_{\mu\nu\rho}$.

3 Numerical results of the form factors in the QCD sum rule approach

Now we are able to calculate the form factors numerically. First, we collect the input parameters used in this work as below [47–49]

$$\begin{aligned}
& m_c(m_c) = 1.275 \pm 0.015\,\text{GeV}\,, \\
& m_s(1\,\text{GeV}) = 142\,\text{MeV}\,, \quad m_u(1\,\text{GeV}) = 2.8\,\text{MeV}\,, \\
& m_d(1\,\text{GeV}) = 6.8\,\text{MeV}\,, \quad \alpha_s(1\,\text{GeV}) = 0.517\,, \\
& m_{J/\psi} = 3.097\,\text{GeV}\,, \quad m_{D^-} = 1.869\,\text{GeV}\,, \\
& m_{D_s^-} = 1.968\,\text{GeV}\,, \quad m_{D^{*-}} = 2.010\,\text{GeV}\,, \\
& m_{D_s^{*-}} = 2.112\,\text{GeV}\,, \quad f_{J/\psi} = 337^{+12}_{-13}\,\text{MeV}\,, \\
& f_{D^-} = 166^{+9}_{-10}\,\text{MeV}\,, \quad f_{D_s^-} = 189^{+9}_{-10}\,\text{MeV}\,, \\
& f_{D^{*-}} = 240^{+10}_{-10}\,\text{MeV}\,, \quad f_{D_s^{*-}} = 262^{+9}_{-12}\,\text{MeV}\,.
\end{aligned} \tag{43}$$

It should be pointed out that the mass of the charm quark used in this work is determined from the charmonium spectrum in [47]. As for the decay constants of the charmed mesons, on the one hand, there is a flood of papers on the theoretical investigation of leptonic decay constants of D^+ and D_s [50–61]; on the other hand, the measurements of decay constants of the pseudoscalar D^+ and D_s mesons have recently been improved by the CLEO and BaBar collaborations [62, 63]. Moreover, the CLEO collaboration reported their work on the value of ratio $f_{D_s^+}/f_{D^+}$ using the measurement of $D_s^+ \to l^+\nu$ channel and obtained $f_{D_s^+} = 274 \pm 13 \pm 7\,\text{MeV}$ [65, 66]. However, the decay constants of D^{*+} and D_s^* mesons have not been directly measured in experiments so far. The only available results on $f_{D^{*+}}$ and $f_{D_s^{*0}}$ from the lattice QCD calculations [53, 61, 64] determine $f_{D_s^*} = 272 \pm 16^{+3}_{-20}\,\text{MeV}$, which is smaller than the value of the decay constant for D_s^+ measured by the CLEO collaboration [65, 66]. To reduce the theoretical uncertainties in the three-point sum rules of the weak transition form factors, due to the quark masses, threshold parameters and Coulomb-like corrections of J/ψ effectively [67], we use the decay constants f_ψ and $f_{D_{d,s}^{(*)-}}$ calculated from the two-point QCD sum rules in leading order of α_s, the same as in the three-point sum rules. The explicit calculations of the decay constants, in the framework of the QCD sum rule approach, for both J/ψ and $D_{d,s}^{(*)}$ are displayed in Appendix C. Our results indicate that $\frac{f_{D_s^*}}{f_{D^*}} \simeq \frac{f_{D_s}}{f_D} = 1.1$, which is in good agreement with the result of a lattice simulation [64] and with experiments [65, 66].

For the threshold parameters s_1^0 and s_2^0, one should determine them by demanding the QCD sum rules results to be relatively stable in the allowed regions for M_1^2 and M_2^2, the values of which should be around the mass square of the corresponding first excited states. As for the heavy-light mesons, the standard value of the threshold in the X channel would be $s_X^0 = (m_X + \Delta_X)^2$, where Δ_X is about 0.6 GeV [68–72], and we simply take it as (0.6 ± 0.1) GeV for the error estimate in the numerical analysis. When it comes to the heavy quarkonium, following the method in [69, 70, 72], we select the effective threshold parameter to ensure the appearance of the pleasant platform and also around the mass square of $\psi(2S)$. In this way, the contributions from both the excited states including $\psi(2S)$ and the continuum states are contained in the spectral function.

3.1 The numerical results of the form factors

3.1.1 Evaluation of the form factors for the $J/\psi \to D^-$ transition

With all the parameters listed above, we can obtain the numerical values of the form factors. The form factors should not depend on the Borel masses M_1 and M_2 in a complete theory. However, as we truncate the operator product expansion up to dimension-5 and keep the perturbative expansion in α_s to leading order, an obvious dependence of the form factors on these two Borel parameters would emerge. Therefore, one should look for a region where the results only mildly vary with respect to the Borel masses, so that the truncation is reasonable and acceptable.

With a careful analysis, $s_1^0 = 13.7\,\text{GeV}^2$ and $s_2^0 = 6.1\,\text{GeV}^2$ are chosen for the calculation of the form factor V. We require the contributions from the higher states to be less than 30% and the value of V does not vary drastically within the selected region for the Borel masses. As commonly understood, the Borel parameters M_1^2 and M_2^2 should not be too large in order to insure that the contributions from the higher excited states and continuum are not too significant. On the other hand, the Borel masses also could not be too small for the sake of validity of OPE in the deep Euclidean region, since the contributions of higher dimension operators pertain to the higher orders in $\frac{1}{M_i}$ $(i = 1, 2)$. Different from that adopted in the previous literature [28, 38], where the ratio of M_1 and M_2 was fixed, in the calculation of the form factors, we let M_1 and M_2 vary independently as suggested by the authors of [41, 73]. In this way, we indeed find a Borel platform $M_1^2 \in [6.0, 10.0]\,\text{GeV}^2$, $M_2^2 \in [1.0, 2.0]\,\text{GeV}^2$, plotted in Fig. 2, which satisfy the conditions discussed above. One can directly read from this figure that $V(q^2 = 0)$ is $0.48^{+0.07}_{-0.05}$, whose uncertainties originate from the variation of the Borel parameters.

Following the same procedure, we also obtain numerical results for the other three form factors A_0, A_1 and A_2 within the chosen Borel window as shown in Fig. 2. The numerical results of the form factors V, A_0, A_1 and A_2 at zero momentum transfer are then

$$V(0) = 0.81^{+0.12}_{-0.08}\,, \qquad A_0(0) = 0.27^{+0.02}_{-0.03}\,,$$
$$A_1(0) = 0.27^{+0.03}_{-0.02}\,, \qquad A_2(0) = 0.34^{+0.07}_{-0.07}\,. \qquad (44)$$

It needs to be emphasized that the form factors $A_1(q^2)$, $A_2(q^2)$ and $A_0(q^2)$ should satisfy the relation $(m_\psi + m_{D^-})A_1(0) + (m_\psi - m_{D^-})A_2(0) = 2m_\psi A_0(0)$ to ensure the disappearance of the divergence at the pole $q^2 = 0$. The theoretical uncertainties in the form factors (44) originate from the Borel masses M_1^2 and M_2^2. They are at the level of 15%, which implies stable results from the QCD sum rules approach.

Indeed there are some extra errors originating from the values of s_1^0 and s_2^0, which correspond to the threshold of the higher excited resonances and continuum states for the J/ψ and D channels, respectively. In the QCD sum rule approach, the values of the threshold parameter usually are in the vicinity of the mass square of the first physical excited state; therefore, we do not investigate the dependence of the form factors on the threshold parameter in this work as in [41, 42], where a larger threshold value of charmonium is adopted. This uncertainty would cause errors in the resultant form factors. Besides, the fluctuations of the charm quark mass can also result in the uncertainties of the form factors, which are evaluated to be at the level of 6%–8%. Moreover, the input parameters such as the decay constants of D meson and J/ψ can also bring about additional uncertainties. Combing the errors from various parameters discussed above, the uncertainties on the form factors can be estimated within 20 to 30%, expected by the general understanding of the theoretical framework.

Next, we can further investigate the q^2 dependence of the form factors V, A_0, A_1 and A_2. The physical region of q^2 for $J/\psi \to D^- l^+ \nu_l$ is $0 \le q^2 \le (m_{J/\psi} - m_{D^-})^2 \simeq 1.5\,\text{GeV}^2$. However, with the QCD sum rules, we could not obtain the form factors in the whole physical region, since the additional singularities – so-called “non-Landau-type” singularities emerge, which have extensively been discussed in [28]. To avoid this kind of singularity, we restrict our calculations to the range of $q^2 \in [0, 0.47]\,\text{GeV}^2$. We show the q^2 dependence of the form factors V, A_0, A_1 and A_2 in Fig. 3.

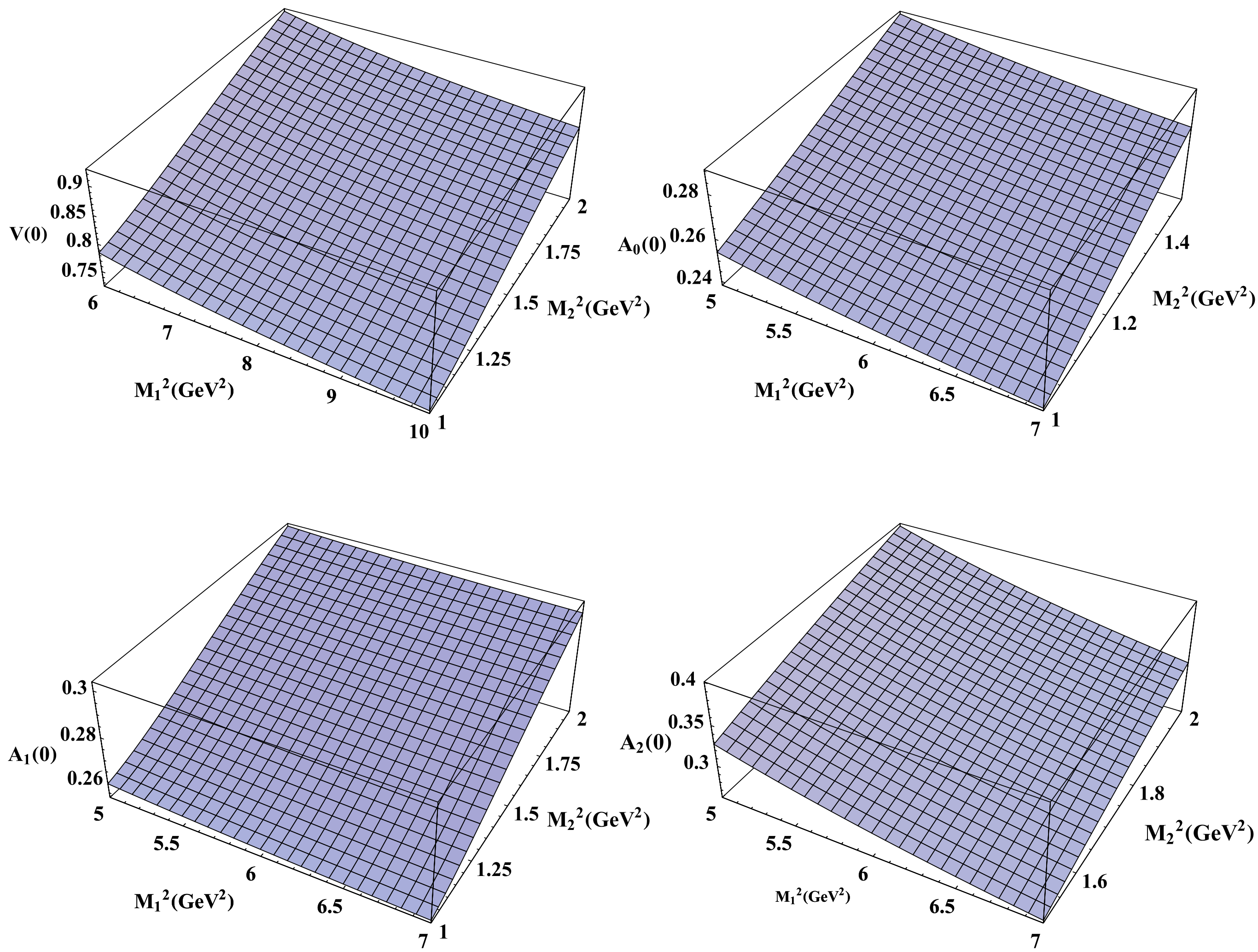

Fig. 2. Dependence of form factors V, A_0, A_1 and A_2 at $q^2 = 0$, responsible for the decay of $J/\psi \to D^-$, on the Borel masses

In addition, for the convenience of applications to phenomenology, one can parameterize the above form factors in a three-parameter form [74]:

$$F_i(q^2) = \frac{F_i(0)}{1 - a_i q^2/m_{D^-}^2 + b_i q^4/m_{D^-}^4}\,, \tag{45}$$

where the F_i denote the form factors V, A_0, A_1 and A_2, and a_i and b_i are the parameters to be fixed. Using the QCD sum rules $F_i(q^2)$ with q^2 restricted within a certain kinematic region, we can fix the parameters a_i and b_i in the expression. This double-pole expression for the form factors can be generalized to the whole kinematic region. Finally, our results for the parameters a_i and b_i are given by

$$\begin{aligned} a_V &= 1.65^{+0.20}_{-0.03}\,, & b_V &= 0.76^{+0.44}_{-0.09}\,,\\ a_{A_0} &= 1.97^{+0.15}_{-0.03}\,, & b_{A_0} &= 1.19^{+0.31}_{-0.05}\,,\\ a_{A_1} &= 0.93^{+0.27}_{-0.12}\,, & b_{A_1} &= 0.46^{+0.29}_{-0.01}\,,\\ a_{A_2} &= 1.47^{+0.14}_{-0.16}\,, & b_{A_2} &= 0.32^{+0.19}_{-0.21}\,. \end{aligned} \tag{46}$$

For the other form factors, which are discussed in the following subsections, we will adopt the same procedure to obtain the form factors in the whole kinematic region.

3.1.2 $J/\psi \to D_s^-$ form factors

Now, we move on to the computations of the form factors for the transition $J/\psi \to D_s^-$, which is quite similar to that for $J/\psi \to D^-$, only with the d quark in D^- being replaced by s. It is also noted that the threshold parameter $s_2^0 = 6.6\ \mathrm{GeV}^2$ for the D_s channel and the Borel window are shifted slightly compared to that of $J/\psi \to D^-$. Since the figures are very similar to the case for $J/\psi \to D^-$, we just omit them. The obtained form factors for $J/\psi \to D_s^-$ at $q^2 = 0$ are

$$\begin{aligned} V(0) &= 1.07^{+0.05}_{-0.02}\,, & A_0(0) &= 0.37^{+0.02}_{-0.02}\,,\\ A_1(0) &= 0.38^{+0.02}_{-0.01}\,, & A_2(0) &= 0.35^{+0.08}_{-0.07}\,. \end{aligned} \tag{47}$$

The parameters a_i and b_i defined above for the q^2 dependence formula (with the replacement of $D \to D_s$) are

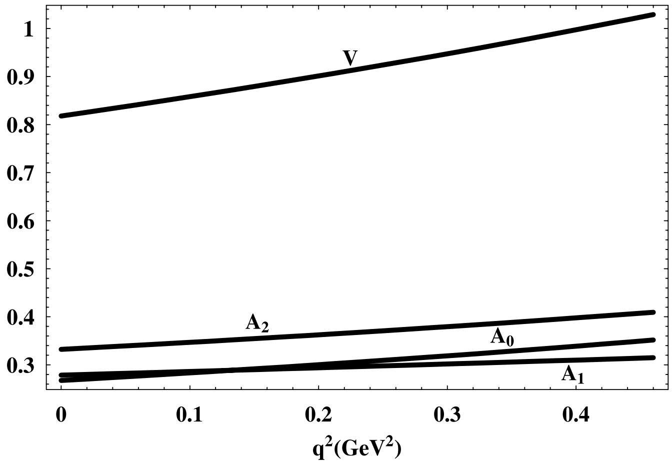

Fig. 3. q^2 dependence of form factors V, A_0, A_1 and A_2 for $J/\psi \to D^-$ within the kinematical region without non-Landau-type singularities

fixed by

$$\begin{aligned} a_V &= 1.86^{+0.26}_{-0.03}\,, & b_V &= 0.90^{+0.43}_{-0.04}\,, \\ a_{A_0} &= 2.12^{+0.0}_{-0.04}\,, & b_{A_0} &= 1.30^{+0.0}_{-0.04}\,, \\ a_{A_1} &= 1.18^{+0.24}_{-0.01}\,, & b_{A_1} &= 0.27^{+0.29}_{-0.04}\,, \\ a_{A_2} &= 1.41^{+0.20}_{-0.29}\,, & b_{A_2} &= 0.38^{+0.15}_{-0.01}\,. \end{aligned} \tag{48}$$

Again, the relation $(m_\psi + m_{D_s})A_1(0) + (m_\psi - m_{D_s})A_2(0) = 2m_\psi A_0(0)$ is well respected, which guarantees that the hadronic matrix element responsible for the $J/\psi \to D_s^-$ transition is divergence free due to the pole at $q^2 = 0$. We show the dependence of the form factors on q^2 in Fig. 4.

3.1.3 $J/\psi \to D^{*-}$ form factors

The evaluation of the form factors responsible for $J/\psi \to D^{*-}$ is performed following the standard procedure, with

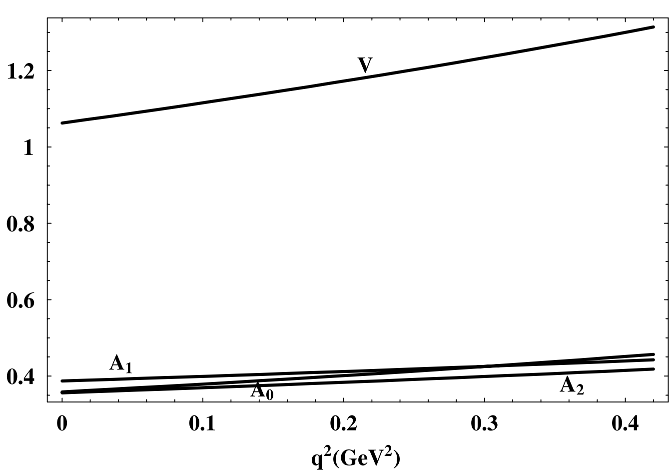

Fig. 4. q^2 dependence of form factors V, A_0, A_1 and A_2 for $J/\psi \to D_s^-$ within the kinematical region without non-Landau-type singularities

appropriate Borel windows obtained. The threshold value for the D^{*-} channel takes $s_2^0 = 6.8\,\text{GeV}^2$ in our numerical analysis. The form factors at zero momentum transfer $q^2 = 0$ are collected below as

$$\begin{aligned} \widetilde{A}_1(0) &= 0.40^{+0.03}_{-0.01}\,, & \widetilde{A}_2(0) &= 0.44^{+0.10}_{-0.04}\,, \\ \widetilde{A}_3(0) &= 0.86^{+0.05}_{-0.01}\,, & \widetilde{A}_4(0) &= 0.91^{+0.06}_{-0.04}\,, \\ \widetilde{V}_1(0) &= 0.41^{+0.01}_{-0.01}\,, & \widetilde{V}_2(0) &= 0.63^{+0.01}_{-0.04}\,, \\ \widetilde{V}_3(0) &= 0.22^{+0.03}_{-0.01}\,, & \widetilde{V}_4(0) &= 0.26^{+0.03}_{-0.05}\,, \\ \widetilde{V}_5(0) &= 1.37^{+0.08}_{-0.03}\,, & \widetilde{V}_6(0) &= 0.87^{+0.05}_{-0.01}\,. \end{aligned} \tag{49}$$

From the above results, we find that the form factors obtained in the QCD sum rule approach respect the relations $\widetilde{A}_1(0) = \widetilde{A}_2(0)$ and $\widetilde{V}_3(0) = \widetilde{V}_4(0)$, which are essential to assure that the hadronic matrix element of $J/\psi \to D^{*-}$ is divergence free at $q^2 = 0$.

Different from that discussed for the $J/\psi \to D^-_{d,s}$ case, not all the form factors that appear in the hadronic matrix element for $J/\psi \to D^{*-}$ are suitably parameterized in the form of (45) with the thee-parameter approximation. To be more specific, the q^2 dependence of the form factors $\widetilde{A}_1(q^2)$ and $\widetilde{A}_2(q^2)$ are written in the following form [75, 76]:

$$F_i(q^2) = \frac{F_i(0)}{\left(1 - a_i q^2/m^2_{D^{*-}}\right)^2}\,, \tag{50}$$

where F_i represents $\widetilde{A}_1$ and $\widetilde{A}_2$, while the other eight form factors are written in the three-parameter form,

$$G_i(q^2) = \frac{G_i(0)}{1 - a_i q^2/m^2_{D^{*-}} + b_i q^4/m^4_{D^{*-}}}\,, \tag{51}$$

where G_i can be $\widetilde{A}_3$, $\widetilde{A}_4$ and $\widetilde{V}_i$ ($i = 1$–6). We then extend the form factors to the whole physical region $0 \le q^2 \le (m_{J/\psi} - m_{D^{*-}})^2 \simeq 1.2\,\text{GeV}^2$, by fitting the parameters by

$$\begin{aligned} a_{\widetilde{A}_1} &= 1.77^{+0.04}_{-0.01}\,, & a_{\widetilde{A}_2} &= 1.95^{+0.17}_{-0.25}\,, \\ a_{\widetilde{A}_3} &= 2.93^{+0.18}_{-0.08}\,, & b_{\widetilde{A}_3} &= 2.47^{+0.54}_{-0.27}\,, \\ a_{\widetilde{A}_4} &= 2.78^{+0.05}_{-0.03}\,, & b_{\widetilde{A}_4} &= 1.78^{+0.27}_{-0.14}\,, \\ a_{\widetilde{V}_1} &= 1.96^{+0.03}_{-0.03}\,, & b_{\widetilde{V}_1} &= 0.98^{+0.07}_{-0.06}\,, \\ a_{\widetilde{V}_2} &= 2.11^{+0.04}_{-0.04}\,, & b_{\widetilde{V}_2} &= 0.21^{+0.05}_{-0.02}\,, \\ a_{\widetilde{V}_3} &= 1.92^{+0.0}_{-0.03}\,, & b_{\widetilde{V}_3} &= 1.87^{+0.12}_{-0.12}\,, \\ a_{\widetilde{V}_4} &= 2.96^{+0.34}_{-0.23}\,, & b_{\widetilde{V}_4} &= 1.97^{+1.03}_{-0.34}\,, \\ a_{\widetilde{V}_5} &= 1.92^{+0.0}_{-0.12}\,, & b_{\widetilde{V}_5} &= 1.03^{+0.0}_{-0.21}\,, \\ a_{\widetilde{V}_6} &= 2.00^{+0.27}_{-0.02}\,, & b_{\widetilde{V}_6} &= 1.08^{+0.53}_{-0.03}\,. \end{aligned} \tag{52}$$

The q^2 dependence of the form factors is shown in Fig. 5.

3.1.4 The form factors for $J/\psi \to D_s^{*-}$

The computation of the amplitude of $J/\psi \to D_s^{*-}$ is almost the same as that for $J/\psi \to D^{*-}$; only the d quark in D^{*-} is replaced by an s quark, with the difference resulting in a different Borel platform. Besides, the threshold parameter for the D_s^{*-} channel is set by $s_2^0 = 7.4\,\text{GeV}^2$ in the

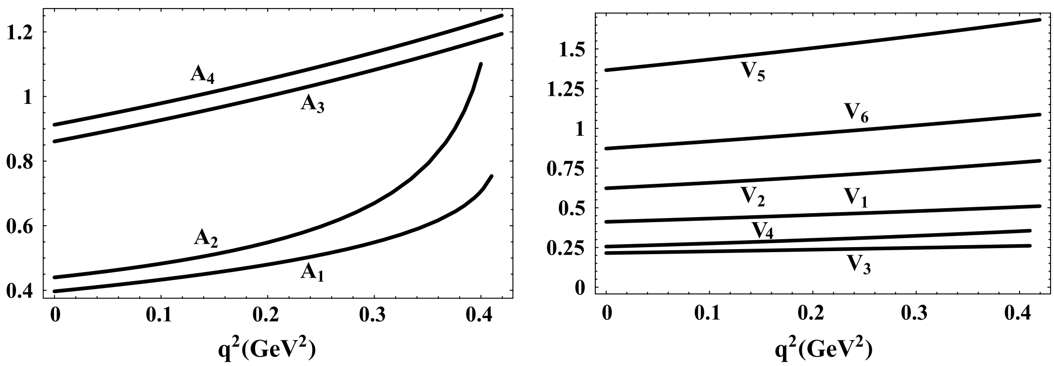

Fig. 5. q^2 dependence of form factors $\widetilde{A}_1$, $\widetilde{A}_2$, $\widetilde{A}_3$, $\widetilde{A}_4$, $\widetilde{V}_1$, $\widetilde{V}_2$, $\widetilde{V}_3$, $\widetilde{V}_4$, $\widetilde{V}_5$ and $\widetilde{V}_6$ for $J/\psi \to D^{*-}$ within the kinematical region without non-Landau-type singularities

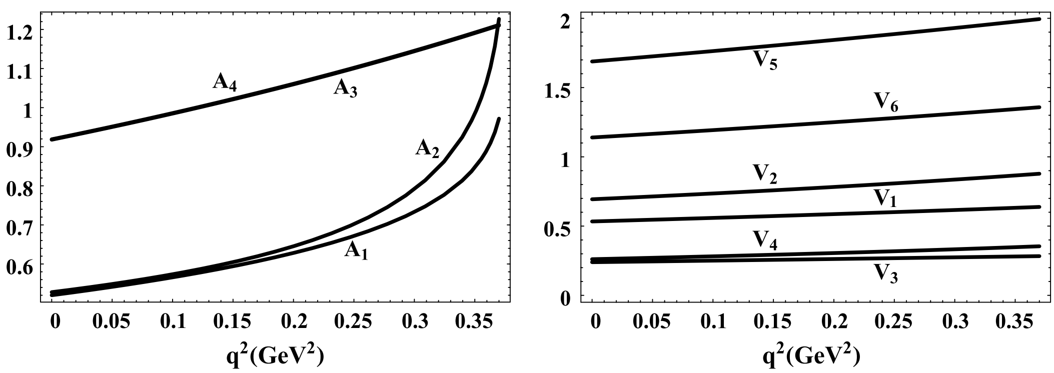

Fig. 6. q^2 dependence of form factors $\widetilde{A}_1$, $\widetilde{A}_2$, $\widetilde{A}_3$, $\widetilde{A}_4$, $\widetilde{V}_1$, $\widetilde{V}_2$, $\widetilde{V}_3$, $\widetilde{V}_4$, $\widetilde{V}_5$ and $\widetilde{V}_6$ for $J/\psi \to D_s^{*-}$ within the kinematical region without non-Landau-type singularities

calculations. The q^2 dependence of the form factors falling into the region of $q^2 \in [0, 0.37]\,\mathrm{GeV}^2$ is plotted in Fig. 6. As mentioned before, the form factors $\widetilde{A}_1$ and $\widetilde{A}_2$ can be parameterized in the form of (50), while the other form factors can be fit in the usual three-parameter form in (51). The parameters a_i and b_i can be determined by reproducing the numbers obtained from the QCD sum rules for the kinematic region $q^2 \in [0, 0.37]\,\mathrm{GeV}^2$ and then we generalize the results to the whole physical region $q^2 \in [0, 0.97]\,\mathrm{GeV}^2$. The values of these parameters together with the form factors at $q^2 = 0$ are collected for convenience as,

$$\begin{aligned}
a_{\widetilde{A}_1} &= 1.92^{+0.0}_{-0.05}\,, & a_{\widetilde{A}_2} &= 1.85^{+0.01}_{-0.21}\,,\\
a_{\widetilde{A}_3} &= 3.07^{+0.12}_{-0.01}\,, & b_{\widetilde{A}_3} &= 1.98^{+0.80}_{-0.16}\,,\\
a_{\widetilde{A}_4} &= 3.08^{+0.06}_{-0.02}\,, & b_{\widetilde{A}_4} &= 2.08^{+0.60}_{-0.26}\,,\\
a_{\widetilde{V}_1} &= 2.05^{+0.13}_{-0.02}\,, & b_{\widetilde{V}_1} &= 0.90^{+0.29}_{-0.06}\,,\\
a_{\widetilde{V}_2} &= 2.53^{+0.06}_{-0.12}\,, & b_{\widetilde{V}_2} &= 0.07^{+0.17}_{-0.35}\,,\\
a_{\widetilde{V}_3} &= 2.04^{+0.16}_{-0.12}\,, & b_{\widetilde{V}_3} &= 2.14^{+0.07}_{-0.0}\,,\\
a_{\widetilde{V}_4} &= 3.32^{+0.37}_{-0.33}\,, & b_{\widetilde{V}_4} &= 1.76^{+1.58}_{-0.67}\,,\\
a_{\widetilde{V}_5} &= 1.92^{+0.22}_{-0.03}\,, & b_{\widetilde{V}_5} &= 0.81^{+0.40}_{-0.05}\,,\\
a_{\widetilde{V}_6} &= 2.00^{+0.26}_{-0.04}\,, & b_{\widetilde{V}_6} &= 0.81^{+0.55}_{-0.15}\,,
\end{aligned} \tag{53}$$

and

$$\begin{aligned}
\widetilde{A}_1(0) &= 0.53^{+0.03}_{-0.01}\,, & \widetilde{A}_2(0) &= 0.53^{+0.05}_{-0.01}\,,\\
\widetilde{A}_3(0) &= 0.91^{+0.05}_{-0.01}\,, & \widetilde{A}_4(0) &= 0.91^{+0.06}_{-0.01}\,,\\
\widetilde{V}_1(0) &= 0.54^{+0.01}_{-0.01}\,, & \widetilde{V}_2(0) &= 0.69^{+0.05}_{-0.06}\,,\\
\widetilde{V}_3(0) &= 0.24^{+0.03}_{-0.01}\,, & \widetilde{V}_4(0) &= 0.26^{+0.03}_{-0.03}\,,\\
\widetilde{V}_5(0) &= 1.69^{+0.10}_{-0.03}\,, & \widetilde{V}_6(0) &= 1.14^{+0.08}_{-0.01}\,.
\end{aligned} \tag{54}$$

In the same way, the relations $\widetilde{A}_1(0) = \widetilde{A}_2(0)$ and $\widetilde{V}_3(0) = \widetilde{V}_4(0)$ are well satisfied.

4 Decay rates for semi-leptonic weak decays of J/ψ

With the form factors derived above, we can perform calculations on the partial widths of the semi-leptonic decays of J/ψ. The relevant CKM parameters are directly taken from the particle data book [48]:

$$|V_{cd}| = 0.2271\,, \quad |V_{cs}| = 0.973\,. \tag{55}$$

For the semi-leptonic decays $J/\psi \to D^{(*)-}_{(d,s)} l^+ \nu_l$ ($l = e, \mu$), the differential partial decay rate is written as

$$\frac{\mathrm{d}\Gamma_{\psi\to D^{(*)-}_{(d,s)} l^+\nu_l}}{\mathrm{d}q^2} = \frac{1}{3}\frac{1}{(2\pi)^3}\frac{1}{32 m_\psi^3} \times \int_{u_{\min}}^{u_{\max}} \left| \widetilde{M}_{\psi\to D^{(*)-}_{(d,s)} l^+\nu_l} \right|^2 \mathrm{d}u\,, \tag{56}$$

where $u = (p_{l^+} + p_D)^2$; p_{l^+} and p_D are the momenta of l^+ and $D^{(*)}_{(d,s)}$ respectively; $|\widetilde{M}|^2$ is the square of the transition amplitude after integrating over the angle between the l^+ and $D^{(*)-}_{(d,s)}$. The transition amplitude $\widetilde{M}$ for $J/\psi \to D^{(*)-}_{(d,s)} l^+ \nu_l$ reads

$$\widetilde{M}_{\psi\to D^{(*)-}_{(d,s)} l^+\nu_l} = \frac{G_\mathrm{F}}{\sqrt{2}} V^*_{cq} \langle D^{(*)}_{(d,s)} | \bar{q}\gamma_\mu (1-\gamma_5) c | J/\psi \rangle \times \bar{\nu}_l \gamma^\mu (1-\gamma_5) l\,. \tag{57}$$

Finally we get the branching ratios of the semi-leptonic decays:

$$\begin{aligned}
\mathrm{BR}(J/\psi \to D^- e^+ \nu_e) &= 7.3^{+4.3}_{-2.2} \times 10^{-12}\,,\\
\mathrm{BR}(J/\psi \to D^- \mu^+ \nu_\mu) &= 7.1^{+4.2}_{-2.2} \times 10^{-12}\,,\\
\mathrm{BR}(J/\psi \to D_s^- e^+ \nu_e) &= 1.8^{+0.7}_{-0.5} \times 10^{-10}\,,\\
\mathrm{BR}(J/\psi \to D_s^- \mu^+ \nu_\mu) &= 1.7^{+0.7}_{-0.5} \times 10^{-10}\,,\\
\mathrm{BR}(J/\psi \to D^{*-} e^+ \nu_e) &= 3.7^{+1.6}_{-1.1} \times 10^{-11}\,,\\
\mathrm{BR}(J/\psi \to D^{*-} \mu^+ \nu_\mu) &= 3.6^{+1.6}_{-1.1} \times 10^{-11}\,,\\
\mathrm{BR}(J/\psi \to D_s^{*-} e^+ \nu_e) &= 5.6^{+1.6}_{-1.6} \times 10^{-10}\,,\\
\mathrm{BR}(J/\psi \to D_s^{*-} \mu^+ \nu_\mu) &= 5.4^{+1.6}_{-1.5} \times 10^{-10}\,,
\end{aligned} \tag{58}$$

where we have combined various uncertainties in the form factors discussed in the previous section to determine the final error tolerance in our theoretical calculations. Our predictions are much below the present experimental upper bounds [2]: $\mathrm{BR}(J/\psi \to D_s^- e^+ \nu_e + \mathrm{c.c.}) < 4.9 \times 10^{-5}$, $\mathrm{BR}(J/\psi \to D^- e^+ \nu_e + \mathrm{c.c.}) < 1.2 \times 10^{-5}$.

A few remarks are in order. First, the sum of the branching fractions of semi-leptonic decays of J/ψ whose final state includes D_s^-, D_s^{*-}, e and μ and their charge conjugate channels can reach values as large as 3.1×10^{-9}, which is expected to be marginally observed at BESIII. Second, it is worthwhile to point out that the decay rates for the dominant semi-leptonic weak decays of J/ψ obtained in [1] were about 7×10^{-9}, which is two times larger than that calculated in this work. This discrepancy can attribute to the heavy-quark spin symmetry and the non-recoil approximation used in [1], also to the different methods used to estimate the non-perturbative form factors.[2] Third, the ratio of $R_1 \equiv \frac{\mathrm{BR}(J/\psi\to D_s^{*-} e^+\nu_e)}{\mathrm{BR}(J/\psi\to D_s^- e^+\nu_e)} \simeq 3.1$ is about 2 times larger than the value calculated in [1], where the assumption of heavy-quark spin symmetry and the non-recoil approximation were adopted. Fourth, the ratios $R_2 \equiv \frac{\mathrm{BR}(J/\psi\to D_s^- e^+\nu_e)}{\mathrm{BR}(J/\psi\to D^- e^+\nu_e)}$ and $R_3 \equiv \frac{\mathrm{BR}(J/\psi\to D_s^{*-} e^+\nu_e)}{\mathrm{BR}(J/\psi\to D^{*-} e^+\nu_e)}$ should be equal to $\left|\frac{V_{cs}}{V_{cd}}\right|^2 \simeq 18.4$ under the SU(3) flavor symmetry limit. Our numerical calculations show that $R_2 \simeq 24.7$ and $R_3 \simeq 15.1$, which implies a large effect of SU(3) symmetry breaking.

[2] In [1], the ISGW model [77] was employed to compute the single form factor η_{12}, while we adopt the QCD sum rule approach to calculate the form factors in this work.

5 Discussion and conclusions

The charmonium J/ψ meson can decay via the strong and electromagnetic interactions; thus, weak decays of J/ψ should be very rare unless there is new physics beyond the standard model to make a substantial contribution. If such weak decays can be measured by the future experiments with sizable branching ratios, it would be a clear signal of new physics.

To make the new physics signal clearly distinguishable from the standard model, a careful study of weak decays in SM is needed. In this work, we calculated the form factors of the weak transitions of $J/\psi \to D^{(*)}_{(d,s)}$ in terms of the QCD sum rules. With the form factors, we estimate the branching ratios of the semileptonic weak decays of J/ψ and find that the sum of the branching ratios corresponding to the dominant modes is about 3.1×10^{-9}, which may be marginally measured by BESIII. The QCD sum rule approach possesses uncontrollable errors, these being as large as 20%–30%, as confirmed by our numerical results. Moreover, due to a Coulomb-type correction in heavy quarkonium decay (or B_c), which may be manifest as the ladder structure in the loop-triangle (as part of multi-loop diagrams), the spectral function needs to be multiplied by a finite renormalization factor [17, 41, 42, 78]. This would bring about another kind of uncertainty. It is expected that this kind of correction can give birth to the double multiplication of the form factors at maximal momentum transfer. However, in this work, we calculate both the three-point QCD sum rules of the weak transition form factors and the two-point sum rules for the decay constant of J/ψ to the same order of α_s. Then it is expected that most uncertainties due to the Coulomb-like corrections are canceled in our calculations; therefore, the Coulomb-like corrections for the J/ψ channel are not included in our calculations. As for the heavy-light mesons, there are no corrections in the power of the inverse velocity for it, since the light quark moves relativistically. Therefore, one should explore the sum rules for both three-point and two-point correlation functions up to next-to-leading order in the strong coupling constant, so that Coulomb-like corrections to the heavy-light mesons can be canceled effectively. Moreover, an explicit calculation of Coulomb-like corrections to the heavy-light vertex in the triangle diagram is still not available now, which has to be left for further considerations.

One can trust the numerical results to a certain accuracy; at least the order of magnitude is reliable. With these form factors we may continue to estimate the rates of non-leptonic weak decays of J/ψ as long as the factorization theorem can be proved to hold. That would be the contents of our next work [79].

The branching ratios of semi-leptonic weak decays of J/ψ are very small in SM, even though their strong decay modes are OZI-suppressed. Our numerical results indicate that even with the large database that will be collected by BESIII, such weak decay modes may only marginally be observed. Therefore, we lay our expectation on our BESIII colleagues and hope they will provide a sufficiently large database to make this challenging field more fruitful.

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Appendix A: The explicit forms of the Wilson coefficients for $\Pi_{\mu\nu}$

In this appendix, we would like to show the explicit expressions of the Wilson coefficients appearing in (11)–(14) after a Borel transformation. As mentioned before, only the perturbative part contributes to the correlation function for the J/ψ decays to $D^-_{d,s}$ at the leading order of the heavy-quark mass expansion and the α_s expansion series. Namely, (10) can be written as

$$f_i = f_i^{\text{pert}}\mathbf{I} + O(\alpha_s) + O(1/m_h)\,, \tag{A.1}$$

with h being the heavy charm quark here. The f_i^{pert} can be related to ρ_i^{pert} defined in (34) by

$$f_i^{\text{pert}} = \int_{(m_c+m_q)^2}^{s_2^0} \mathrm{d}s_2 \int_{s_1^{\mathrm{L}}}^{s_1^0} \mathrm{d}s_1 \frac{\rho_i^{\text{pert}}\left(s_1,s_2,q^2\right)}{\left(s_1-p_1^2\right)\left(s_2-p_2^2\right)}\,, \tag{A.2}$$

or

$$\widehat{\mathcal{B}} f_i^{\text{pert}} = \int_{(m_c+m_q)^2}^{s_2^0} \mathrm{d}s_2 \int_{s_1^{\mathrm{L}}}^{s_1^0} \mathrm{d}s_1 \frac{1}{M_1^2}\mathrm{e}^{-s1/M_1^2}\frac{1}{M_2^2}\mathrm{e}^{-s_2/M_2^2} \times \rho_i{}^{\text{pert}}\left(s_1,s_2,q^2\right). \tag{A.3}$$

The lowest bound of s_1, i.e., s_1^{L} can be determined by (31) to be

$$\begin{aligned} s_1^{\mathrm{L}} = -\frac{1}{2m_q^2}\Bigg[& m_c^4 - \left(2m_q^2+s_2+q^2\right)m_c^2 + m_q^2 \\ &+ s_2q^2 - m_q^2\left(s_2+q^2\right) \\ &+ \sqrt{m_c^4 - 2\left(m_q^2+s_2\right)m_c^2 + \left(m_q^2-s_2\right)^2} \\ &\times\sqrt{m_c^4 - 2\left(m_q^2+q^2\right)m_c^2 + \left(m_q^2-q^2\right)^2}\Bigg]\,, \end{aligned} \tag{A.4}$$

according to the Landau equation [80][3]

The obvious forms of ρ_i^{pert} $(i=0,2,4,5)$ are

$$\begin{aligned}
\rho_0^{\text{pert}}\left(s_1,s_2,q^2\right) =& -\frac{3}{4\pi^2\lambda^{3/2}} \\
&\times\left[m_c\lambda + (m_c-m_q)s_1\left(2m_c^2-2m_q^2-s_1+s_2+q^2\right)\right], \\
\rho_2^{\text{pert}}\left(s_1,s_2,q^2\right) =& -\frac{3s_1}{2\pi^2\lambda^{5/2}} \\
&\times\left\{(m_c-m_q)s_1\left[6m_c^4 - 6\left(2m_q^2+s_1-s_2\right)m_c^2\right.\right. \\
&\left. + 6m_q^4 + (s_1-s_2)^2 + 6m_q^2(s_1-s_2)\right] \\
&+ m_c[2(m_c-m_q)(2m_c+m_q) - s_1+s_2]\lambda \\
&+ q^2\left[2(m_c-m_q)s_1\left(3m_c^2-3m_q^2-s_1+2s_2\right)\right. \\
&\left.\left. + m_c\lambda + (m_c-m_q)s_1q^2\right]\right\}, \\
\rho_4^{\text{pert}}\left(s_1,s_2,q^2\right) =& \frac{3}{4\pi^2\lambda^{5/2}} \\
&\times\left\{\left[-m_c\lambda^2 + (m_c-m_q)\left(2s_2m_c^2 + s_1\left(2m_q^2+s_1-s_2\right)\right)\right]\lambda\right. \\
&+ 2(m_c-m_q)s_1\left[3(s_1+s_2)m_c^4\right. \\
&- 2\left(3(s_1+s_2)m_q^2 + (s_1-s_2)(s_1+2s_2)\right)m_c^2 \\
&\left. + (s_1-s_2)^2s_2 + 3m_q^4(s_1+s_2) + 2m_q^2(s_1-s_2)(s_1+2s_2)\right] \\
&+ (m_c-m_q)q^2\left[2s_1\left(s_2^2 + \left(-2m_c^2+2m_q^2+s_1\right)s_2\right.\right. \\
&\left. + \left(m_c^2-m_q^2\right)\left(-3m_c^2+3m_q^2+4s_1\right)\right) \\
&\left.\left. - \left(2m_c^2+s_1\right)\lambda - 4s_1\left(m_c^2-m_q^2+s_2\right)q^2\right]\right\}, \\
\rho_5^{\text{pert}}\left(s_1,s_2,q^2\right) =& \frac{3}{8\pi^2\lambda^{3/2}} \\
&\times\left\{\lambda\left(m_qs_1+m_cs_2-m_cq^2\right)\right. \\
&- 2(m_c-m_q)\left[\lambda m_c^2 + \left(m_c^2-m_q^2\right)s_1\left(m_c^2-m_q^2-s_1+s_2\right)\right. \\
&\left.\left. + s_1\left(m_c^2-m_q^2+s_2\right)q^2\right]\right\}.
\end{aligned} \tag{A.5}$$

In this appendix, we adopt the notion $\lambda \equiv \lambda(s_1,s_2,q^2)$ for convenience.

Appendix B: The expressions of the Wilson coefficients for $\Pi_{\mu\nu\rho}$

Similarly, we will display the forms of the Wilson coefficients emerging in (19)–(28) after performing a Borel transformation. As has been discussed in the text, only the perturbative part contributes to the three-point function,

$$F_i = F_i^{\text{pert}}\mathbf{I} + O(\alpha_s) + O(1/m_h)\,. \tag{B.1}$$

The relation between F_i^{pert} and $\rho_i'^{\text{pert}}$ is given by

$$F_i^{\text{pert}} = \int_{(m_c+m_q)^2}^{s_2^0} \mathrm{d}s_2 \int_{s_1^{\mathrm{L}}}^{s_1^0} \mathrm{d}s_1 \frac{\rho_i'^{\text{pert}}\left(s_1,s_2,q^2\right)}{\left(s_1-p_1^2\right)\left(s_2-p_2^2\right)} \tag{B.2}$$

or

$$\widehat{\mathcal{B}} F_i^{\text{pert}} = \int_{(m_c+m_q)^2}^{s_2^0} \mathrm{d}s_2 \int_{s_1^{L}}^{s_1^0} \mathrm{d}s_1 \frac{1}{M_1^2}\mathrm{e}^{-s1/M_1^2}\frac{1}{M_2^2}\mathrm{e}^{-s_2/M_2^2} \times \rho_i'^{\text{pert}}\left(s_1,s_2,q^2\right), \tag{B.3}$$

[3] For a review, see [80].

where the definition of $s_1^{\rm L}$ is given in (A.4). The explicit expressions of $\rho_i^{\prime\rm pert}$ ($i=1,4,5,6,7,9,11,12,14,15$) are displayed here:

$$
\begin{aligned}
\rho_1^{\prime\rm pert}(s_1,s_2,q^2) &= \frac{3}{4\pi^2\lambda^{5/2}} \\
&\times \{-4s_1^2m_c^4+(8m_q^2s_1^2+(s_1-s_2)(s_1+s_2)^2 \\
&-(s_1-3s_2)\lambda)m_c^2+2m_q(s_1+s_2)\lambda m_c-4m_q^4s_1^2 \\
&+s_2(3s_1+s_2)(\lambda-(s_1-s_2)^2)-m_q^2(s_1+s_2)(s_1^2-s_2^2+\lambda) \\
&+q^2[4s_1m_c^4+(3s_2^2+10s_1s_2-s_1(8m_q^2+s_1)+\lambda)m_c^2 \\
&-2m_q\lambda m_c+3s_2(s_1+s_2)^2+4m_q^4s_1-s_2\lambda \\
&+m_q^2(s_1^2-10s_1s_2-3s_2^2+\lambda) \\
&+(m_c^2-m_q^2+s_2)q^2(-s_1-3s_2+q^2)]\}\,,
\end{aligned}
$$

$$
\begin{aligned}
\rho_4^{\prime\rm pert}(s_1,s_2,q^2) &= \frac{3s_1}{4\pi^2\lambda^{5/2}} \\
&\times \{-4s_2m_c^4-2(-4s_2m_q^2+s_1^2+3s_2^2-4s_1s_2-\lambda)m_c^2 \\
&+4m_qm_c\lambda-4m_q^4s_2-(s_1+3s_2)((s_1-s_2)^2-\lambda) \\
&+2m_q^2(s_1^2-4s_1s_2+3s_2^2-\lambda)+q^2(4m_c^4+4(s_2-2m_q^2)m_c^2 \\
&+4m_q^4+3(s_1+s_2)^2-4m_q^2s_2-\lambda \\
&+q^2(2m_c^2-2m_q^2-3s_1-s_2+q^2))\}\,,
\end{aligned}
$$

$$
\begin{aligned}
\rho_5^{\prime\rm pert}(s_1,s_2,q^2) &= \frac{1}{4\pi^2\lambda^{5/2}} \\
&\times \{-(s_1+s_2-q^2)[3(m_c^2-m_q^2+s_2)q^4 \\
&-6((2s_1+s_2)m_c^2+s_2(s_1+s_2)-m_q^2(2s_1+s_2))q^2 \\
&-3(2s_1m_c^4-(4s_1m_q^2+3s_1^2+s_2^2-\lambda)m_c^2+2m_qm_c\lambda \\
&+2m_q^4s_1+m_q^2(3s_1^2+s_2^2-\lambda)+s_2(s_1^2-s_2^2+\lambda))] \\
&-2s_2[3s_1(-2(m_c^2-m_q^2)^2+s_1^2-s_2^2-4(m_c^2-m_q^2)s_2) \\
&-3(2m_c^2+s_1)\lambda+3s_1q^2(q^2-2(s_1+s_2))]\}\,,
\end{aligned}
$$

$$
\begin{aligned}
\rho_6^{\prime\rm pert}(s_1,s_2,q^2) &= \frac{3}{4\pi^2\lambda^{5/2}} \\
&\times \{-2s_1(3s_1+s_2)m_c^4+2[s_1(2m_q^2+s_1-s_2)(3s_1+s_2) \\
&-(2s_1+s_2)\lambda]m_c^2-4m_qs_1m_c\lambda \\
&+s_1[-2(3s_1+s_2)m_q^4+2(-3s_1^2+2s_1s_2+s_2^2+\lambda)m_q^2 \\
&+(s_1-s_2)^2(s_1+s_2)-(s_1+3s_2)\lambda] \\
&+q^2[2s_1m_c^4+2(\lambda-2s_1(m_q^2+2s_1))m_c^2 \\
&+s_1(2m_q^4+8s_1m_q^2-(s_1+s_2)(3s_1+5s_2)+\lambda) \\
&+s_1(2m_c^2-2m_q^2+3s_1+5s_2-q^2)q^2]\}\,,
\end{aligned}
$$

$$
\begin{aligned}
\rho_7^{\prime\rm pert}(s_1,s_2,q^2) &= -\frac{3}{16\pi^2\lambda^{5/2}} \\
&\times \{(2(m_c-m_q)^2-2s_2)\lambda^2-2[s_2(-s_1+s_2-q^2) \\
&+(m_c^2-m_q^2)(s_1+s_2-q^2)](s_1-s_2+q^2)\lambda \\
&-8[(m_c^2-m_q^2+s_2)(s_1+s_2-q^2)-2s_1s_2] \\
&\times [((s_1+s_2-q^2)^2-4s_1s_2)m_c^2+s_1(m_c^2-m_q^2+s_2)^2 \\
&+s_1^2s_2-s_1(m_c^2-m_q^2+s_2)(s_1+s_2-q^2)]\}\,,
\end{aligned}
$$

$$
\begin{aligned}
\rho_9^{\prime\rm pert}(s_1,s_2,q^2) &= -\frac{3}{8\pi^2\lambda^{5/2}} \\
&\times \{-((m_c-m_q)^2-s_2)\lambda^2-[s_2(-s_1+s_2-q^2) \\
&+(m_c^2-m_q^2)(s_1+s_2-q^2)](s_1-s_2+q^2)\lambda \\
&+2[\lambda m_c^2+(m_c^2-m_q^2)s_1(m_c^2-m_q^2-s_1+s_2) \\
&+s_1(m_c^2-m_q^2+s_2)q^2]\lambda \\
&-4[(m_c^2-m_q^2+s_2)(s_1+s_2-q^2)-2s_1s_2] \\
&\times [((s_1+s_2-q^2)^2-4s_1s_2)m_c^2+s_1(m_c^2-m_q^2+s_2)^2 \\
&+s_1^2s_2-s_1(m_c^2-m_q^2+s_2)(s_1+s_2-q^2)]\}\,,
\end{aligned}
$$

$$
\begin{aligned}
\rho_{11}^{\prime\rm pert}(s_1,s_2,q^2) &= -\frac{3s_1}{8\pi^2\lambda^{5/2}} \\
&\times \{-\lambda^2-(-2m_c^2+2m_q^2+s_1-s_2-q^2) \\
&\times [-2(m_c-m_q)^2-s_1+s_2+q^2]\lambda \\
&-4[((s_1+s_2-q^2)^2-4s_1s_2)m_c^2+s_1(m_c^2-m_q^2+s_2)^2 \\
&+s_1^2s_2-s_1(m_c^2-m_q^2+s_2)(s_1+s_2-q^2)] \\
&\times (-2m_c^2+2m_q^2+s_1-s_2-q^2)\}\,,
\end{aligned}
$$

$$
\begin{aligned}
\rho_{12}^{\prime\rm pert}(s_1,s_2,q^2) &= -\frac{3}{8\pi^2\lambda^{5/2}} \\
&\times \{s_1\lambda^2-s_1(-2m_c^2+2m_q^2+s_1-s_2-q^2)(s_1+s_2-q^2)\lambda \\
&+2[\lambda m_c^2+(m_c^2-m_q^2)s_1(m_c^2-m_q^2-s_1+s_2) \\
&+s_1(m_c^2-m_q^2+s_2)q^2]\lambda \\
&-4s_1[((s_1+s_2-q^2)^2-4s_1s_2)m_c^2+s_1(m_c^2-m_q^2+s_2)^2 \\
&+s_1^2s_2-s_1(m_c^2-m_q^2+s_2)(s_1+s_2-q^2)] \\
&\times (-2m_c^2+2m_q^2+s_1-s_2-q^2)\}\,,
\end{aligned}
$$

$$
\begin{aligned}
\rho_{14}^{\prime\rm pert}(s_1,s_2,q^2) &= -\frac{3}{4\pi^2\lambda^{7/2}} \\
&\times \{-2m_c^2(s_1+s_2-q^2)\lambda^2 \\
&-s_1[4s_1s_2(-2m_c^2+2m_q^2+s_1-s_2-q^2) \\
&-(s_2(-s_1+s_2-q^2)+(m_c^2-m_q^2)(s_1+s_2-q^2)) \\
&\times (s_1+s_2-q^2)+3(m_c^2-m_q^2+s_2)(s_1+s_2-q^2) \\
&\times (2m_c^2-2m_q^2-s_1+s_2+q^2)]\lambda \\
&+4s_1[4s_2\{(s_1+s_2-q^2)^2+s_1s_2\}s_1^2 \\
&-3(m_c^2-m_q^2+s_2)\{(s_1+s_2-q^2)^2+6s_1s_2\} \\
&\times (s_1+s_2-q^2)s_1+2(m_c^2(s_1+s_2-q^2)^4 \\
&+\{6s_1(m_c^2-m_q^2+s_2)^2-2m_c^2s_1s_2\}(s_1+s_2-q^2)^2 \\
&+2s_1^2s_2(3(m_c^2-m_q^2+s_2)^2-4m_c^2s_2))-2(m_c^2-m_q^2+s_2) \\
&\times \{3((s_1+s_2-q^2)^2-4s_1s_2)m_c^2 \\
&+5s_1(m_c^2-m_q^2+s_2)^2\}(s_1+s_2-q^2)]\}\,,
\end{aligned}
$$

$$
\begin{aligned}
\rho_{15}^{\prime\rm pert}(s_1,s_2,q^2) &= -\frac{3}{4\pi^2\lambda^{7/2}} \\
&\times \{-2m_c^2(s_1+s_2-q^2)\lambda^2 \\
&-s_1(4s_1s_2(-2m_c^2+2m_q^2+s_1-s_2-q^2) \\
&-[s_2(-s_1+s_2-q^2)+(m_c^2-m_q^2)(s_1+s_2-q^2)] \\
&\times (s_1+s_2-q^2)+3(m_c^2-m_q^2+s_2)(s_1+s_2-q^2) \\
&\times (2m_c^2-2m_q^2-s_1+s_2+q^2))\lambda \\
&+4[-10s_2^2(s_1+s_2-q^2)s_1^3 \\
&+12s_2(m_c^2-m_q^2+s_2)((s_1+s_2-q^2)^2+s_1s_2)s_1^2 \\
&-3(2s_2((s_1+s_2-q^2)^2-4s_1s_2)m_c^2
\end{aligned}
$$

$$+\left(m_c^2-m_q^2+s_2\right)^2\left(\left(s_1+s_2-q^2\right)^2+6s_1s_2\right)\right)\left(s_1+s_2-q^2\right)s_1+2\left(m_c^2-m_q^2+s_2\right)\left(m_c^2(s_1+s_2-q^2\right)^4+2s_1\left(\left(m_c^2-m_q^2+s_2\right)^2-m_c^2s_2\right)\left(s_1+s_2-q^2\right)^2+2s_1^2s_2\left(\left(m_c^2-m_q^2+s_2\right)^2-4m_c^2s_2\right)\right)\Big]\Big\}\,. \quad \text{(B.4)}$$

Appendix C: Decay constants of J/ψ and $D_{d,s}^{(*)}$ in two-point QCD sum rule approach

In this appendix, we would like to collect the sum rules for the decay constants of J/ψ and $D_{d,s}^{(*)}$ for the completeness of the paper. The decay constant of J/ψ in the two-point QCD sum rule approach can be written as [69, 82]

$$f_\psi^2 m_\psi^2 \mathrm{e}^{-\frac{m_\psi^2}{M^2}} = \int_{4m_c^2}^{s_\psi^0} \mathrm{d}s \frac{s}{4\pi^2}\left(1-\frac{4m_c^2}{s}\right)^{1/2}\left(1+\frac{2m_c^2}{s}\right)\mathrm{e}^{-\frac{s}{M^2}} + \left[-\frac{m_c^2}{4M^2}+\frac{1}{16}+\frac{1}{48}\mathrm{e}^{-\frac{4m_c^2}{M^2}}\right]\langle 0|\frac{\alpha_s}{\pi}G_{\mu\nu}^2|0\rangle\,, \quad \text{(C.1)}$$

where the non-relativistic approximation for the gluon condensate has been adopted for the convenience of performing a Borel transformation. It is observed that the gluon condensate has a tiny effect on the results of the form factors and hence these are neglected in the sum rules of the charmed mesons. The non-perturbative condensates used in the evaluation of the sum rules can be grouped as

$$\begin{aligned}
\langle 0|\bar{q}q|0\rangle &= -(1.65\pm 0.15)\times 10^{-2}\,\mathrm{GeV}^3 \quad (q=u,d)\,,\\
\langle 0|\bar{s}s|0\rangle &= (0.8\pm 0.1)\langle 0|\bar{q}q|0\rangle\,,\\
\langle 0|\frac{\alpha_s}{\pi}G_{\mu\nu}^2|0\rangle &= 0.005\pm 0.004\,\mathrm{GeV}^2\,,\\
\langle 0|\bar{q}_i\sigma\cdot Gq_i|0\rangle &= m_0^2\langle 0|\bar{q}_iq_i|0\rangle\,, \quad \text{(C.2)}
\end{aligned}$$

where $m_0^2=(0.8\pm 0.2)\,\mathrm{GeV}^2$ and the subscript "i" denotes the flavor of the quarks. Based on the two-point sum rules of J/ψ and the parameters shown above, we can derive the decay constant of J/ψ as 337_{-13}^{+12} MeV, where we have combined the uncertainties from the variations of the Borel masses and the threshold value for the J/ψ channel.

The sum rules for the decay constant of D_q can be given by [82–86]

$$\begin{aligned}
&\frac{m_{D_q}^4}{(m_c+m_q)^2}f_{D_q}^2\mathrm{e}^{-\frac{m_{D_q}^2}{M^2}}\\
&= \frac{3}{8\pi^2}\int_{(m_c+m_q)^2}^{s_{D_q}^0}\mathrm{d}s\left[1-\frac{(m_c-m_q)^2}{s}\right]\lambda^{1/2}\left(s,m_c^2,m_q^2\right)\mathrm{e}^{-\frac{s}{M^2}}\\
&+\left(-m_c+\frac{m_q}{2}+\frac{m_qm_c^2}{2M^2}\right)\mathrm{e}^{-\frac{m_c^2}{M^2}}\langle 0|\bar{q}q|0\rangle\\
&-\frac{m_c}{2M^2}\left(1-\frac{m_c^2}{2M^2}\right)\mathrm{e}^{-\frac{m_c^2}{M^2}}\langle 0|\bar{q}\sigma\cdot Gq|0\rangle\,, \quad \text{(C.3)}
\end{aligned}$$

from which we can arrive at the decay constants of the pseudoscalar charmed mesons: $f_{D_d}=166_{-10}^{+9}$ MeV and $f_{D_s}=189_{-10}^{+9}$ MeV.

The decay constants of the charmed vector mesons $f_{D_q^*}$ in the framework of the QCD sum rule approach can be calculated to be [82, 85, 87]

$$\begin{aligned}
&f_{D_q^*}^2 m_{D_q^*}^2\mathrm{e}^{-\frac{m_{D_q^*}^2}{M^2}}\\
&= \frac{1}{8\pi^2}\int_{(m_c+m_q)^2}^{s_{D_q^*}^0}\mathrm{d}s\lambda^{1/2}\left(s,m_c^2,m_q^2\right)\\
&\times\left[2-\frac{m_c^2+m_q^2-6m_cm_q}{s}-\frac{\left(m_c^2-m_q^2\right)^2}{s^2}\right]\mathrm{e}^{-\frac{s}{M^2}}\\
&+\left\{\left[-\left(m_c+\frac{8}{3}m_q\right)+\frac{1}{2}\frac{m_qm_c^2}{M^2}\right]\mathrm{e}^{-\frac{m_c^2}{M^2}}\right.\\
&\left.+\frac{2m_q\left(4M^2-m_c^2\right)}{3M^2}\right\}\langle 0|\bar{q}q|0\rangle\\
&+\frac{m_c^3}{4M^4}\mathrm{e}^{-\frac{m_c^2}{M^2}}\langle 0|\bar{q}\sigma\cdot Gq|0\rangle\,, \quad \text{(C.4)}
\end{aligned}$$

from which we can achieve the decay constants of the charmed vector mesons: $f_{D_d^*}=240_{-10}^{+10}$ MeV and $f_{D_s^*}=262_{-12}^{+9}$ MeV.

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